K – Stage CFAR Detection in Binomial Distribution Pulse Jamming

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1 Introduction
The priority of a sequential detector over a conventional detector is in the reduction of radar energy at radar target detection. Target detection is usually in all range channels for fixed azimuth. The effectiveness of a sequential detector is defined as a ratio between the fixed sample number in the conventional Neyman-Pearson procedure and the average sample number in the K-stage procedure. The comparison between these detectors is made to achieve the same false alarm and detection probability.

An important step in the creation of an efficient approach to optimisation and analysis of truncated sequential procedures was made by Sosulin and Gavrilov in [7], within the framework of the study of K-stage procedures of a statistical hypotheses test. Questions concerning the application of these procedures and a method of their optimisation to various binary detection problems and multi-alternative detection problems were investigated in [5,7]. These articles illustrate the universality and efficiency of the developed approach for the solution of various truncated sequential detection problems and show the possibility to design K-stage signal detectors that significantly surpass in efficiency wide-spread detectors with fixed sample size. The effectiveness of multi-channel processing is defined through Monte Carlo computer simulation.

The performances of some sequential CFAR detectors are proposed and evaluated in [6]. The observations are passed through a dead-zone limited. Numerical results show a significant reduction of the average number of observations needed to achieve the same false alarm and detection probability as compared to a fixed-sample-size CFAR detector using the same kind of test statistic.

We study the efficiency of K-stage CFAR BI processors in conditions of binomial distributed pulse jamming. The speed-up of a K-stage CFAR BI processor towards a conventional CFAR BI processor is obtained. We assume that the noise in the test cell is Rayleigh envelope distributed and the target returns are fluctuating according to Swelling II model, as in [3,4].

The effectiveness of a K-stage test is not dependent on the increasing of the probability for the appearance of pulse jamming. It is so because the detection probability is constant. In order to keep the detection probability constant, the signal-to-noise ratio increases with the increase of the probability for the appearance of pulse jamming.

The computational speed-up of the multi-channel procedure depends on the presence of a signal and the level of the BI detection threshold. We recommend the K-stage procedure to be used with a higher BI detection threshold in conditions of few targets and with a lower BI detection threshold in conditions of many targets. Research is performed in MATLAB environment.

2. Signal model.
Let us assume that \( L \) pulses hit the target, which is modeled according to Swerling case II. The received signal is sampled in range by using \((N+1)\) resolution cells resulting in a matrix with \((N+1)\) rows and \( L \) columns. Each column of the data matrix consists of the values of the signal.

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1This work is supported by IIT – 010044, MPS Ltd. Grant “RDR”, Bulgarian NF “SR” Grant № I – 902/99 and Ministry of Defense Grant № 20/02.
obtained for $L$ pulse intervals in one range resolution cell. Let us also assume that the first $(N/2)$ and the last $(N/2)$ rows of the data matrix are used as a reference window in order to estimate the "noise-plus-interference" level in the test resolution cell of the radar. In this case the samples of the reference cells result in a matrix $X$ of the size $NL$. The test cell or the radar target includes the elements of the $(N/2+1)$ row of the data matrix and is a vector $X_0$ of length $L$.

The sampling rate is such that the samples are statistically independent. After filtration the signal is applied to a square-law detector and then processed in the CA CFAR decision element. In conditions of binomial distribution of pulse jamming, the background environment includes the interference-plus-noise situation, which may appear at the output of the receiver with the probability $2e(1-\varepsilon)$, interference-plus-noise situation with the probability $\varepsilon^2$ and the noise only situation with the probability $(1-\varepsilon)^2$, where $\varepsilon = 1 - \sqrt{1 - F}$, $F$ is the average repetition frequency of PJ and $T_r$ is the length of pulse transmission [8]. We assume that in these situations the outputs of the reference window are observations from statistically independent exponential random variables. Consequently, the probability density function (pdf) of the reference window outputs may be defined by:

$$f(x_i) = \frac{(1-\varepsilon)^2}{\lambda_0} \exp\left(-\frac{x_i}{\lambda_0}\right) + 2\varepsilon(1-\varepsilon) \exp\left(-\frac{x_i}{\lambda_0(1+r)}\right) + \frac{\varepsilon^2}{\lambda_0(1+2r)} \exp\left(-\frac{x_i}{\lambda_0(1+2r)}\right), \quad i = 1, \ldots, NL$$

where $\lambda_0$ is the average power of the receiver noise, $r_j$ is the average interference-to-noise ratio (INR) of pulse jamming and $NL$ is the number of the reference cells. The sample from the test resolution cell $x_{0l}$ is assumed to be distributed according to Swerling II case with the pdf given by:

$$f(x_{0l}) = \frac{(1-\varepsilon)^2}{\lambda_0(l+s)} \exp\left(-\frac{x_{0l}}{\lambda_0(l+s)}\right) + 2\varepsilon(1-\varepsilon) \exp\left(-\frac{x_{0l}}{\lambda_0(l+r+s)}\right) + \frac{\varepsilon^2}{\lambda_0(l+2r+s)} \exp\left(-\frac{x_{0l}}{\lambda_0(l+2r+s)}\right), \quad l = 1, \ldots, L$$

where $s$ is the per pulse average signal-to-noise ratio (SNR).

3. CFAR BI detectors.

We study a K-stage CFAR BI detector and compare it with a conventional CFAR BI detector in our work. For that purpose we have offered and used the detection probability of a CFAR BI detector for the determination of the scale factor of CFAR detectors in K-stage and BI detectors.

Binary integration processors are used for target radar detection (BI-processors) and employ a two-step threshold technique [3]. In the first step a preliminary decision is made about each pulse of the train reflected from the target. Pulse detection is declared if the first adaptive threshold is exceeded in the cell under test. In a conventional mean level constant false alarm rate (CA CFAR) detector proposed by Finn and Johnson in [1], the detection threshold is determined as a product of the noise level estimate in the reference window and a scale factor to achieve the design probability of false alarm. In the second step the number of decisions where the first threshold is exceeded is counted and this number is compared with the second (BI detector) threshold. Target radar detection is declared if the second threshold is exceeded.

All samples from the test resolution cell at the input of the binary integrator $x_{0l}$ $(l = 0, \ldots, L)$ are compared with the first (CFAR detector) threshold $H_{DI}$ according to the rule [3]:

$$\Phi_l(x_{0l}) = \begin{cases} 1, & \text{if } x_{0l} \geq H_{DI} \\ 0, & \text{otherwise} \end{cases}$$

The thresholds of CFAR detectors are: $H_{DI} = TV_l$, where $V_l$ is the noise level estimate in the $l$ channel, $T$ is the scale factor which keeps the false alarm probability constant.

The binary integrator performs summing of $L$ values of $\Phi_l$. Target radar detection is declared if this sum exceeds the second threshold $M$. 

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The probability of target detection for a CA CFAR BI processor in the presence of binomial distribution pulse jamming is obtained in [4]:

\[
P_d = \sum_{i=0}^{L} C_i^l P_d^i(1 - P_d)^{L-i}
\]

where \( P_d \) is the detection probability of a CA CFAR processor in the presence of binomial distribution \( PJ [4] \).

The probability of false alarm is evaluated by (5), setting \( s = 0 \).


In this section, we describe a sequential K-stage CFAR detector using the test statistic of a CA CFAR BI detector. The test statistic is a sum of the values from the output of a CA CFAR BI detector. To describe the test, we define the output of a CFAR detector as (3). The test statistic to be used is

\[
S(x_{(k)}) = \sum_{i=1}^{k} \Phi_i
\]

where \( k \) is the number of observations. We have used \( x_{(k)} \) to represent \((x_{01}, \ldots, x_{0k})\).

We propose the use of a K-stage sequential CFAR detector. It compares the test statistic with the bounder \( B \) (fig.1) if the outcome of \( k \) is less than \( L \) \((L \text{ is fixed})\) and if \( k \) is bigger than \((L - M + 1)\). It compares the test statistic to a threshold \( M \) if the outcome to \( k \) is bigger than \( M \) and if \( k \) is less than \( L \). Specifically, the test for the detector at the \( k \)-th observation is as follows:

\[
K_M = M \quad \text{and} \quad K_B = L - M
\]

if \( M < (L/2) \) then \( K_K = (K_M + 1) \quad \text{else} \quad K_K = (K_B + 1) \)

if \( k < K_K \) then \( S(x_{(k)}) = \sum_{i=1}^{k} \Phi_i \)

if \( k \geq K_K \) and \( K_M > k > K_B \) test \( S(x_{(k)}) < B, \quad \text{say} \ H_0 \)

\[
\text{otherwise, continue}
\]

if \( k \geq K_K \) and \( K_B > k > K_M \) test \( S(x_{(k)}) > M, \quad \text{say} \ H_1 \)

\[
\text{otherwise, continue}
\]
The boundaries for (8 and 9) are depicted on Fig.1.

\[ \sum_{i=1}^{k} \Phi_i (13) \]

\[ \text{if } k \geq (K_M \text{ and } K_B) \text{ and } k < L \text{ test } S(x(k)) \begin{cases} > M, & \text{say } H_1 \\ < B, & \text{say } H_0 \\ \text{otherwise, continue} & \end{cases} \]

\[ \text{if } k = L \text{ test } S(x(k)) \begin{cases} \geq M, & \text{say } H_1 \\ \text{otherwise, say } H_0 & \end{cases} \]

The boundaries for (8 and 9) are depicted on Fig.1.

\[ \sum_{i=1}^{k} \Phi_i (14) \]

\[ \text{if } L \leq k \leq M/k \text{ if } k > M \]

\[ \text{if } L \leq k \leq M/k \text{ if } k > M \]

The efficiency of the K-stage CFAR procedure will be estimated toward the CFAR BI procedure. The radar efficiency factor of the sequential procedure is:

\[ \mu = \frac{L}{k} \]

where \( L \) is the full sample number in the BI procedure, \( \bar{k} \) is the average sample number in the K-stage procedure.

5. Estimation the parameters of the a K-stage CFAR processor

Fig.2. Block diagram of a K-stage sequential CFAR detector

(SLD – Square Law Detector)
When the detection algorithm uses all sample sells \((L)\), i.e. a CFAR BI detector is used, the following number of computational steps is needed:

1. Comparing: 
   \((L+1)\)  
2. Assignment: 
   \((LN+3L+1)\)  
3. Summarizing: 
   \((LN+L)\)  
4. Multiplication: 
   \((L)\)

If we assume that these operations are carried out for equal periods of time \(T=1\) or for one computational step, then the total number of computational steps is:

\[
T_0^{(1)} = 2LN + 6L + 2
\]

When a \(K\)-stage algorithm is used, the following number of computational steps is needed:

1. Comparing:
   \((k - K_k + 1)(4 + (k + K_k)/2)\)
2. Assignment:
   \((kN + (k - K_k + 1)(k + K_k)/2)\)
3. Summarizing:
   \((kN + (k - K_k + 1)(2 + 3(k + K_k)/2)\)
4. Multiplication:
   \((k - K_k + 1)(k + K_k)/2\)

The total number of computational steps is:

\[
T_0 = 2kN + 3(k - K_k + 1)(k + K_k + 2)
\]

Consequently, when a \(K\)-stage algorithm is used, target detection is speeded up with:

\[
K_{up1} = \frac{T_0^{(1)}}{T_0}
\]

where \(K_{up1}\) is the speed-up of the computational process for a single-stage detection procedure.

The computational speed-up of a multi-channel procedure is:

\[
K_{up} = \frac{K_{up1}(G - g) + K_{up1}g}{G}
\]

where \(G\) is the number of channels (the number of the algorithm tests), \(g\) is the number of the targets in the channels.


In this paper we investigate the influence of the effectiveness of the \(K\)-stage procedure over the probability for the appearance of PJ for \(SNR=0\) and \(SNR\) (\(P_D=0.9\)). In order to keep the detection probability constant \((0.9)\), the signal-to-noise ratio must be increased when the probability for the appearance of pulse jamming increases (Table 1).

<table>
<thead>
<tr>
<th>(M)</th>
<th>2</th>
<th>10</th>
<th>14</th>
</tr>
</thead>
<tbody>
<tr>
<td>(P_J)</td>
<td>0</td>
<td>0.1</td>
<td>0.2</td>
</tr>
<tr>
<td>2</td>
<td>22.3</td>
<td>68.2</td>
<td>69</td>
</tr>
<tr>
<td>10</td>
<td>21.47</td>
<td>33.1</td>
<td>29.64</td>
</tr>
<tr>
<td>14</td>
<td>21.46</td>
<td>26.88</td>
<td>25</td>
</tr>
</tbody>
</table>

Table 1. Signal-to-noise ratio needed to keep \(P_D=0.9\), for probability for the appearance of pulse jamming from 0 to 0.9 and threshold \(M=2, 10\) and 14.
The experimental results are obtained for the following parameters: average power of the receiver noise $\lambda_n = 1$, average interference-to-noise ratio (INR) $r_j = 30\, \text{dB}$, probability for the appearance of PJ from 0 to 0.9, number of reference cells $N = 16$, number of azimuth channels $L = 16$, probability of false alarm $P_{fa} = 10^{-6}$ and signal-to-noise ratio SNR = 0 and SNR($P_0 = 0.9$). The thresholds $M$ of the K-stage CFAR processor receive values 2, 10 and 14.

The experimental results from Fig.3 and 4 reveal that there is small influence of the probability for the appearance of PJ over radar effectiveness, because the needed signal-to-noise ratio increases with the increase of PJ. When the threshold $M$ of the K-stage CFAR detector is small (2) and the signal-to-noise ratio is big, the effectiveness is about 4–5. For signal-to-noise ratio SNR=0 this detector is not effective. When the BI threshold $M$ of the sequential detector is 14 and the signal-to-noise ratio is SNR=0, the effectiveness is about 3+4, but for signal-to-noise ratio SNR($P_0 = 0.9$) this detector is not effective.

The computational speed-up of a single-stage detection procedure $K_{up1}$ for the case of PJ is:

\[
\begin{array}{|c|c|c|}
\hline
\text{SNR}=0 & M = 2 & M = 10 & M = 14 \\
\hline
\text{SNR} (P_0 = 0.9) & 4.76 & 0.8 & 0.51 \\
\hline
\end{array}
\]

Table 2. Speed-up of a single-stage detection procedure $K_{up1}$

In practice a few number of the radar cells are full with the target signal. Most of the sample cells are full with background environment. For example, when we have one target in the radar cells ($g = 1$) and the number of the channels is $G = 1000$, the computational speed-up of the multi-channel procedure $K_{up}$ is:

\[
\begin{array}{|c|c|c|}
\hline
M = 2 & M = 10 & M = 14 \\
\hline
0.514 & 1.499 & 2.658 \\
\hline
\end{array}
\]

Table 3. Speed-up of a multi-channel procedure $K_{up}$.

Because of the achieved results, we recommend the K-stage procedure to be used with a higher detection threshold $M$ in conditions of few targets and with a low $M$ in conditions of many targets.

7. Conclusions.

We propose and evaluate the performance of K–stage constant false alarm rate (CFAR) detectors in binomial distribution pulse jamming. By using this approach, the time of detection of radar (target) signals is minimised or the observation time is economised. Computational speed-up of the signal processing is achieved. We use the analytical characteristics of CA CFAR BI detectors in pulse jamming achieved in [3, 4] for the calculation of the two thresholds of the K-stage detector. The number of stages, or the radar efficiency, is achieved by using Monte-Karlo simulation.

We recommend the K-stage procedure to be used with a higher detection threshold $M$ in conditions of few targets and with a low $M$ in conditions of many targets. The speed-up of the single-stage and the multi-channel procedures are equal. Research is performed in MATLAB environment.

Acknowledgement. We are grateful to Professor Y. Sosulin for working together.

REFERENCES
