CFAR PI DETECTOR IN PRESENCE OF BINOMIAL DISTRIBUTION FLOW FROM RANDOMLY ARRIVING IMPULSE INTERFERENCE

I. G. Garvanov

(Submitted by Academician Ivan Popchev on February 17, 2005)

Abstract
Post-detection integration constant false alarm rate (CFAR PI) detector in presence of flow from randomly arriving impulse interferences with binomial distribution is studied in this paper. The detection probability and the false alarm probability of the CFAR PI detector in these interferences are offered. The probability characteristics of the research CFAR PI detector in presence of binominal distribution flow are more general and include the probability characteristics of this detector in case of Poisson distribution flow from impulse interference. The obtained analytical results of the CFAR PI detector can be used in both, radar and communications receivers.

Key words: Radar Detection, CFAR Detector, Randomly Arriving Impulse Interference, Average Decision Threshold (ADT), Probability of Detection, Probability of False Alarm.

1. Introduction. Radar target detection in noise, clutter or interference is a very important device in each radar receiver. In modern radar, the received signal is sampled in range by range resolution cells. The detection target signal is declared when the received echo signal amplitude exceed a detection threshold. The adaptive detection threshold is obtained by scaling the noise level estimate with a scale factor $T$ to achieve the design probability of false alarm $P_{FA}$. The noise level in the test cell is estimated by averaging the outputs of the nearby resolution cells. This is the conventional cell averaging constant false alarm rate (CA CFAR) pulse detector proposed by Finn and Johnson [1]. The main aim of all detectors is to maximize the target detection probability.

In the post-detection integration (PI) CFAR detector, comparing the non-coherent integrated signal to a adaptive threshold makes the final decision about the presence of a target. The probability of false alarm of the post detection integrator (PI) is extremely sensitive to randomly arriving impulse interference, which is undesirable in radar systems. The interference destroys the visibility of the radar. Random interfering pulses with an average repetition frequency about 0.1-0.2 forms the Poisson distributed flow from impulse interference [2]. Unfortunately, the law of distribution changes from Poisson to binomial with the increasing of the probability for the appearance of impulse interference. The change of distribution law and/or parameters of impulse interference makes impossible the keeping of constant false alarm rate and leads to decreasing of the probability of detection.

The effectiveness of the CFAR PI detector in presence of Poisson distributed flow from pulse jamming and Raleigh amplitude distribution in both test and reference windows are studied by Behar in [3]. The mathematical model of this interference is proposed by Akimov in [4]. The PI detector in presence of some randomly arriving impulse interference with Raleigh amplitude distribution, only in reference window, is researched by Himonas in [5]. In this paper, we consider here more complex and more general situation than the one studied in [3, 5]. We consider the case when the CFAR PI detector works in presence of binomial distributed flow from pulse interference and Raleigh amplitude distribution. In our research, we assume that the target returns according to the Swerling II case [6, 7].
The mathematical expressions for calculating the probability of detection and the probability of false alarm of CFAR PI detector in binomial distributed randomly arriving impulse interference are offered. From received analytical equations for the probability characteristics of the CFAR PI detector in binomial distribution impulse interference easy can be obtained detection and false alarm probability of this detector in Poisson distribution flow from impulse interference. The new analytical equations are more general and described the probability characteristics of research detector for binomial and Poisson distribution impulse interferences. The reason is that both distributions are connected.

2. Signal model. In a CFAR detector, the square-law detected received signal is sampled in range and in time. The sampled signal is stored in the input detector memory like a data matrix. Let us assume that \( L \) pulses hit the target which is modeled according to Swerling case II. The received signal is sampled in range by using “\(N+1\)” resolution cells resulting in a matrix with “\(N+1\)” rows and “\(L\)” columns. Each column of the data matrix consists of the values of the signal obtained for “\(L\)” pulse intervals in one range resolution cell. Let us also assume that the first “\(N/2\)” and the last “\(N/2\)” rows of the data matrix are used as reference cells in order to estimate the “noise-plus-interference” level in the test resolution cell of the radar. In this case, the samples of the reference cells result in a matrix \( X \) of the size “\(NxL\)”. The test cell or the radar target image includes the elements of the “\(N/2+1\)” row of the data matrix and is a vector \( X_0 \) of the length \( L \). In conditions of binomial distribution of pulse jamming, the background environment includes the interference-plus-noise situation, which may appear at the output of the receiver with the probability \( 2\varepsilon(1-\varepsilon) \), two interference-plus-noise situation with the probability \( (1-\varepsilon)^2 \), where \( \varepsilon = 1 - \sqrt{1 - t_c F} \), \( F \) is the average repetition frequency of impulse interference and \( t_c \) is the length of impulse transmission \(^1\). The distribution is binomial when the probability of impulse interference is above 0.1-0.2 \(^2\). In these situations, the outputs of the reference window are observations from statistically independent exponential random variables. Consequently, the probability density function (pdf) of the reference window outputs may be defined by

\[
f(x_n) = \frac{(1-\varepsilon)^2}{\lambda_0} \exp\left(\frac{-x_n}{\lambda_0}\right) + 2\varepsilon(1-\varepsilon) \exp\left(\frac{-x_n}{\lambda_0(1+r_j)}\right) + \varepsilon^2 \exp\left(\frac{-x_n}{\lambda_0(1+2r_j)}\right) \quad n = 1,...,N \quad i = 1,...,L
\]

where \( \lambda_0 \) is the average power of the receiver noise and \( r_j \) \( / \lambda_0 \) is the per pulse average interference-to-noise ratio (INR) of pulse interference. The set of samples from the test resolution cell \( \{x_{li}\}_{l} \) is assumed to be distributed according to Swerling II case with the pdf given by

\[
f(x_l) = \frac{(1-\varepsilon)^2}{\lambda_0(l+s)} \exp\left(\frac{-x_{li}}{\lambda_0(l+s)}\right) + 2\varepsilon(1-\varepsilon) \exp\left(\frac{-x_{li}+r_j}{\lambda_0(l+r_j+s)}\right) + \varepsilon^2 \exp\left(\frac{-x_{li}+2r_j}{\lambda_0(l+2r_j+s)}\right) \quad l = 1,...,L \quad s \text{ is the per pulse average signal-to-noise ratio.}
\]

When the probability for appearance of impulse interference is small (to 0.1), then \( \varepsilon^2 \equiv 0 \) and the flow is Poisson distributed \(^1\).

3. CFAR Post-detection Integration (PI) Detector. The CFAR PI detector consists of a square-law envelope detector (SLD), linear post detection integrator, the noise level estimator and the comparator \(^{3,5}\). The estimate of the noise level in this detector is formed as a sum of the samples from the reference window: \( V = \sum_{l=1}^{L} \sum_{n=1}^{N} x_{li} \).

Let \( q_o \) to be the signal formed by summing the elements of the test resolution cell \( q_o = \sum_{l=1}^{L} x_{li} \). Then the target is detected according to the following algorithm
where $H_I$ is the hypothesis that the test resolution cell, i.e., $X_0$, contains the echoes from the target and $H_0$ is the hypothesis that the test resolution cell, i.e., $X_0$, contains the receiver noise only. The constant $T_{p_1}$ is a scale coefficient, which is determined in order to maintain a given constant false alarm rate. The probability of target detection is determined as $[6]

\begin{equation}
 P_D = P(q_0 > T_{p_1}V | H_1) = \int_0^{T_{p_1}V} P_V(V) dV \int P_{q_0}(q_0 | H_1) dq_0
\end{equation}

where $P_V(V)$ is the pdf of the noise level estimate and $P_{q_0}(q_0 | H_1)$ is the conditional pdf of the test summed signal under hypothesis $H_1$. The probability of false alarm is determined by substituting $s=0$ in (4).

4. Analysis of the CFAR PI detector. In the case of non-adaptive integration, the size of the reference window “$N x L$” and the size of the test cell “$L$” are nonrandom variables. The probability density function $P_V(V)$ of the noise level estimate, under hypothesis $H_1$ is found as inverse Laplace transform of the moment generating function (mgf) of the random variable $V$: $P_V(V) = L^{-1}[M_V(U)]$. In this case, the mgf of the estimate $V$ is defined by $M_V(U) = M_s(U)$, where $M_s(U)$ is the mgf of the random variable $x_{ln}$: $M_s(U) = \int \exp(-Ux)f(x)dx$, defined by $[8]$.

\begin{equation}
 M_V(U) = \frac{(1-\varepsilon)^2}{1 + U\lambda_0} + \frac{2\varepsilon(1-\varepsilon)}{1 + U\lambda_0(1 + r_j)} + \frac{\varepsilon^2}{1 + U\lambda_0(1 + 2r_j)}
\end{equation}

Then the mgf of the estimate $V$ takes the form

\begin{equation}
 M_V(U) = \sum_{i=1}^{LN} C_i LN^i e^{LN-i} \sum_{j=0}^{LN-i-j} C_j LN^{-i-j} \frac{1}{(1 + \lambda^*_0 U)} \frac{1}{(1 + \lambda_0^* U)} \frac{1}{(1 + \lambda^*_0 U)} \frac{1}{(1 + \lambda_0^* U)} \frac{1}{(1 + \lambda^*_0 U)} \frac{1}{(1 + \lambda_0^* U)} \frac{1}{(1 + \lambda^*_0 U)} \frac{1}{(1 + \lambda_0^* U)}
\end{equation}

where $\lambda^*_0 = \lambda_0(1 + r_j)$ and $\lambda_0^* = \lambda_0(1 + 2r_j)$.

By using the inverse Laplace transform of the moment generating function of the estimate $V$, we obtained the probability density function of this estimate

\begin{equation}
 P_V(V) = A \left[ P_1(V) + P_2(V) + P_3(V) \right]
\end{equation}

where

\begin{align}
 A &= \sum_{i=1}^{LN} C_i LN^i e^{LN-i} \sum_{j=0}^{LN-i-j} C_j LN^{-i-j} \frac{1}{(1 + \lambda^*_0 U)} \frac{1}{(1 + \lambda_0^* U)} \frac{1}{(1 + \lambda^*_0 U)} \frac{1}{(1 + \lambda_0^* U)} \frac{1}{(1 + \lambda^*_0 U)} \frac{1}{(1 + \lambda_0^* U)} \frac{1}{(1 + \lambda^*_0 U)} \frac{1}{(1 + \lambda_0^* U)}

 P_1(V) &= \sum_{a=0}^{L-1} V^a \exp(-V / \lambda_0) Q_1

 P_2(V) &= \sum_{a=0}^{L-1} V^a \exp(-V / \lambda_0^*) Q_2

 P_3(V) &= \sum_{a=0}^{L-1} V^a \exp(-V / \lambda^*_0) Q_3
\end{align}

and

\begin{align}
 Q_1 &= (-1)^{L-1} \sum_{b=0}^{L-1} \left[ i-1 + b \right] \left[ LN - i - a - b - 2 \right] \left( \lambda_0^* \right)^{L-1-a-b} \left( \lambda^*_0 \right)^{L-1-a-b} \left( \lambda_0 \right)^{L-1-a-b} \left( \lambda^*_0 \right)^{L-1-a-b}

 Q_2 &= (-1)^{L-1} \sum_{b=0}^{L-1} \left[ i-1 + b \right] \left[ LN - j - a - b - 2 \right] \left( \lambda_0^* \right)^{L-1-a-b} \left( \lambda^*_0 \right)^{L-1-a-b} \left( \lambda_0 \right)^{L-1-a-b} \left( \lambda^*_0 \right)^{L-1-a-b}

 Q_3 &= (-1)^{L-1} \sum_{b=0}^{L-1} \left[ i-1 + b \right] \left[ LN - j - a - b - 2 \right] \left( \lambda_0^* \right)^{L-1-a-b} \left( \lambda^*_0 \right)^{L-1-a-b} \left( \lambda_0 \right)^{L-1-a-b} \left( \lambda^*_0 \right)^{L-1-a-b}
\end{align}
The moment generating function of the random variable \(q_0\) is found identically as mgf of \(V\)

\[
M_{q_0}(U) = \sum_{n=0}^{L} C_L^n \sum_{m=0}^{L-n} C_L^m \left[2\epsilon(1-\epsilon)\right]^n \left(1-\epsilon\right)^{2(L-n-m)} \frac{1}{(1+\lambda^*_U)^n(1+\lambda^*_U)^{L-n-m}}
\]

where: \(\lambda = \lambda_0(1+s), \lambda^*_U = \lambda_0(1+s+r), \lambda^*_U = \lambda_0(1+s+2r)\).

By using the inverse Laplace transform of the moment generating function of the random variable \(q_0\), we obtained the probability density function \(P_{q_0}(q_0 / H_i)\)

\[
P_{q_0}(q_0 / H_i) = B[P_1(q_0) + P_2(q_0) + P_3(q_0)]
\]

where

\[
B = \sum_{n=0}^{L} C_L^n \sum_{m=0}^{L-n} C_L^m \left[2\epsilon(1-\epsilon)\right]^n \left(1-\epsilon\right)^{2(L-n-m)}
\]

\[
P_1(q_0) = \sum_{c=0}^{n-1} \frac{q_0^n \exp(-q_0 / \lambda^*_U)}{c!(\lambda^*_U)^{c+1}} Q_1^c
\]

\[
P_2(q_0) = \sum_{c=0}^{n-1} \frac{q_0^n \exp(-q_0 / \lambda^*_U)}{c!(\lambda^*_U)^{c+1}} Q_1^c
\]

\[
P_3(q_0) = \sum_{c=0}^{L-m-n-1} \frac{q_0^n \exp(-q / \lambda^*_U)}{c!(\lambda^*_U)^{c+1}} Q_3^c
\]

and

\[
Q_1^c = (-1)^{L-c} \sum_{d=0}^{L-n-1} \binom{n-1+d}{d} \left(\frac{\lambda_0}{\lambda^*_U}\right)^L \left(\frac{\lambda^*_U}{\lambda_0}\right)^{-L-n-1} \left(\frac{\lambda^*_U}{\lambda_0}\right)^{L-n-m-1}
\]

\[
Q_2^c = (-1)^{L-c} \sum_{d=0}^{m-1} \binom{m-1+d}{d} \left(\frac{\lambda_0}{\lambda^*_U}\right)^{-L-n-m} \left(\frac{\lambda^*_U}{\lambda_0}\right)^{L-n-m}
\]

The probability of detection of the CFAR PI detector, as given in eqn. (4) is

\[
P_D = A.B \int [P_1(V) + P_2(V) + P_3(V)] dV \int_{VT_{PI}} P_1(q_0) dq_0 + \int_{VT_{PI}} P_2(q_0) dq_0 + \int_{VT_{PI}} P_3(q_0) dq_0
\]

or

\[
P_D = A.B \int [P_1(V) + P_2(V) + P_3(V)][I_1(V) + I_2(V) + I_3(V)] dV
\]

where

\[
I_1(V) = \sum_{c=0}^{n-1} \frac{Q_1^c \sum_{i=0}^{n-1} \left(VT_{PI}\lambda^*_U\right)^i}{i!} \frac{\exp(-VT_{PI} / \lambda^*_U)}{l!}
\]

\[
I_2(V) = \sum_{c=0}^{n-1} \frac{Q_2^c \sum_{i=0}^{n-1} \left(VT_{PI}\lambda^*_U\right)^i}{i!} \frac{\exp(-VT_{PI} / \lambda^*_U)}{l!}
\]

\[
I_3(V) = \sum_{c=0}^{L-m-n-1} \frac{Q_3^c \sum_{i=0}^{n-1} \left(VT_{PI}\lambda^*_U\right)^i}{i!} \frac{\exp(-VT_{PI} / \lambda^*_U)}{l!}
\]

By calculating of the integrals we obtained the detection probability of the CFAR PI detector in presence of flow from binomial distribution impulse interference.

\[
P_D = A.B((F_1 + F_2 + F_3 + ....... + F_9))
\]

where
The probability of false alarm of the CFAR PI detector is evaluated by (17), setting $s=0$. When the probability for appearance of impulse interference is small (to 0.1) then the flow is Poisson distributed and the probability characteristics of the CFAR PI detector are obtained as in [3]. The characteristics of research detectors for binomial distribution impulse interference are more general and include the probability characteristics of these detectors in presence of Poisson distribution impulse interference. If in (17), we accept that $2r=0$, $\varepsilon^2=0$, $2\varepsilon (1-\varepsilon)\rightarrow \varepsilon_0$ and $(1-\varepsilon)^2\rightarrow (1-\varepsilon_0)$, we obtained the probability of detection of the CFAR PI detector in presence of Poisson distribution impulse interference as in [3]. Where $\varepsilon_0$ is probability for appearance of impulse interference and $(1-\varepsilon_0)$ is probability for noise only.

5. Results. The new analytical expressions obtained in the previous section make possible the estimation of the quality of CFAR PI detectors in the presence of flow from very intensive impulse interference with binomial distributed. We obtained numerical and simulation results to illustrate the performance of the CFAR PI detector. The influence of the impulse interference parameters over detection probability and the average decision threshold (ADT) of the CFAR PI detector are investigated in this paper.

In Fig. 1 are shown the detection probability of the CFAR PI detector for different values of the probability for appearance of impulse interference (0.1, 0.5 and 0.9), when the average interference-to-noise ratio INR = 10 and 30dB. The results are obtained for the following input parameters: average power of the receiver noise $\lambda_0=1$; number of reference cells $N=16$, number of test cells $L=16$ and probability of false alarm $P_{FA}=10^{-3}$. The detection probability decrease with increase of the average interference-to-noise ratio and the probability for appearance of impulse interference for fixed SNR (Fig.1).

The average decision threshold of the CFAR PI detector is shown on Fig. 2. The experimental results are obtained for the following input parameters: average power of the receiver noise $\lambda_0=1$; average interference-to noise ratio (INR) $r=10$ and 30dB; probability for the appearance of impulse interference from 0.1 to 0.9; number of reference cells $N=16$. 

\[
\begin{align*}
F_1 &= \sum_{c=0}^{m-1} \sum_{i=1}^{c} Q_i \sum_{j=0}^{l} \left( \frac{\lambda_0 T_{pr}}{\lambda_i + \lambda_0 T_{pr}} \right)^{a+l} \sum_{a=0}^{l} Q_j \left( \frac{\lambda_0}{\lambda_i + \lambda_0 T_{pr}} \right)^{a+l} \\
F_2 &= \sum_{c=0}^{m-1} \sum_{i=1}^{c} Q_i \sum_{j=0}^{l} \left( \frac{\lambda_0^2 T_{pr}}{\lambda_i + \lambda_0^2 T_{pr}} \right)^{a+l} \sum_{a=0}^{l} Q_j \left( \frac{\lambda_0^2}{\lambda_i + \lambda_0^2 T_{pr}} \right)^{a+l} \\
F_3 &= \sum_{c=0}^{m-1} \sum_{i=1}^{c} Q_i \sum_{j=0}^{l} \left( \frac{\lambda_0 T_{pr}}{\lambda_i + \lambda_0 T_{pr}} \right)^{a+l} \sum_{a=0}^{l} Q_j \left( \frac{\lambda_0}{\lambda_i + \lambda_0 T_{pr}} \right)^{a+l} \\
F_4 &= \sum_{c=0}^{m-1} \sum_{i=1}^{c} Q_i \sum_{j=0}^{l} \left( \frac{\lambda_0^2 T_{pr}}{\lambda_i + \lambda_0^2 T_{pr}} \right)^{a+l} \sum_{a=0}^{l} Q_j \left( \frac{\lambda_0^2}{\lambda_i + \lambda_0^2 T_{pr}} \right)^{a+l} \\
F_5 &= \sum_{c=0}^{m-1} \sum_{i=1}^{c} Q_i \sum_{j=0}^{l} \left( \frac{\lambda_0^3 T_{pr}}{\lambda_i + \lambda_0^3 T_{pr}} \right)^{a+l} \sum_{a=0}^{l} Q_j \left( \frac{\lambda_0^3}{\lambda_i + \lambda_0^3 T_{pr}} \right)^{a+l} \\
F_6 &= \sum_{c=0}^{m-1} \sum_{i=1}^{c} Q_i \sum_{j=0}^{l} \left( \frac{\lambda_0^4 T_{pr}}{\lambda_i + \lambda_0^4 T_{pr}} \right)^{a+l} \sum_{a=0}^{l} Q_j \left( \frac{\lambda_0^4}{\lambda_i + \lambda_0^4 T_{pr}} \right)^{a+l} \\
F_7 &= \sum_{c=0}^{m-1} \sum_{i=1}^{c} Q_i \sum_{j=0}^{l} \left( \frac{\lambda_0^5 T_{pr}}{\lambda_i + \lambda_0^5 T_{pr}} \right)^{a+l} \sum_{a=0}^{l} Q_j \left( \frac{\lambda_0^5}{\lambda_i + \lambda_0^5 T_{pr}} \right)^{a+l} \\
F_8 &= \sum_{c=0}^{m-1} \sum_{i=1}^{c} Q_i \sum_{j=0}^{l} \left( \frac{\lambda_0^6 T_{pr}}{\lambda_i + \lambda_0^6 T_{pr}} \right)^{a+l} \sum_{a=0}^{l} Q_j \left( \frac{\lambda_0^6}{\lambda_i + \lambda_0^6 T_{pr}} \right)^{a+l} \\
F_9 &= \sum_{c=0}^{m-1} \sum_{i=1}^{c} Q_i \sum_{j=0}^{l} \left( \frac{\lambda_0^7 T_{pr}}{\lambda_i + \lambda_0^7 T_{pr}} \right)^{a+l} \sum_{a=0}^{l} Q_j \left( \frac{\lambda_0^7}{\lambda_i + \lambda_0^7 T_{pr}} \right)^{a+l}
\end{align*}
\]
number of test cells \( L = 16 \) and probability of false alarm \( P_{FA} = 10^{-3} \). The results for the ADT are received using the signal to noise ratio needed to keep the detection probability equal to 0.5 (SNR \( (P_D = 0.5) \)). The results for ADT are obtained by numerical calculation (17) and Monte Carlo simulation in MATLAB environment (results of \( 10^4 \) independent experiments). The results obtained by both methods are identical.

When the INR and the probability for the appearance of impulse interference increase then the average decision threshold also increase. The ADT increases slowly with increases of the probability for appearance of impulse interference. The ADT of the researched detector is equal of INR of the random arriving impulse interference for the all interval of appearance of the interference (0.1 to 0.9).

6. Conclusions. This paper has studied the problem of the CFAR PI detector designed to operate in an interference-saturated environment. Unfortunately, the law of distribution of randomly arriving impulse interference changes from Poisson to binomial with the increasing of the probability for the appearance of the impulse interference. The change of the distribution law and parameters of randomly arriving pulses interference makes impossible the keeping of constant false alarm rate and leads to decreasing of the probability of detection.

We have proposed and analyzed the performance of the CFAR PI detector in strong impulse interference. The mathematical expressions for calculating of the detection probability and false alarm probability of this detector in the presence of flow from randomly arriving impulses interference with binomial distribution are obtained. The obtained analytical results for the research CFAR PI detector are more general and include the probability characteristics of this detector in presence of Poisson distribution impulse interference.

The obtained equations may be successfully applied for target detection in existing radar and communication networks by using pulse train signals.

REFERENCES


Institute of Information Technologies
Bulgarian Academy of Sciences
Akad. G. Bonchev Str., bl. 2,
1113 Sofia, Bulgaria
E-mail: igarvanov@iit.bas.bg