Доклади на Българската академия на науките
Comptes rendus de l'Académie bulgare des Sciences

Tome 63, No 7, 2010

MATHEMATIQUES

Sous-ensembles flous

FUZZY MULTI-CRITERIA DECISION MAKING ALGORITHMS

Vania Peneva, Ivan Popchev

(*Submitted on February 22, 2010*)

Mini Review

Abstract

The paper presents a brief description of algorithms for solving fuzzy multi-criteria decision making problems. The evaluations of the alternatives by the criteria can be real numbers by different scales, fuzzy preference relations by each criterion or trapezoidal fuzzy numbers by the criteria. The weights of the criteria can be real numbers, weighting functions or fuzzy relation between the couples of criteria according to their importance. Different combinations between the initial information may be used to solve the problems of choice or ordering of the alternatives.

Key words: multicriteria decision making, fuzzy relations, trapezoidal fuzzy numbers, aggregation operators, fuzzy relations' properties

2000 Mathematics Subject Classification: 03E72

1. Introduction. The fuzzy sets theory provides a useful way to approach multi-criteria decision making (MCDM) problems. The new topic of the decision making – fuzzy multi-criteria decision making (FMCDM) uses imprecise and fuzzy data. The considered FMCDM algorithms use the following initial information: a finite set of alternatives, among which a decision maker has to choose (choice problem) or to rank (ranking problem); a finite set of judges or criteria on the basis of which the alternatives are evaluated; a criteria importance, i.e. weights of the criteria significances.

This work is supported by the Bulgarian Academy of Sciences under grant 010093/04.02.2010.

The alternatives in decision making problems are usually evaluated from different points of views that correspond to particular criteria. The criteria can be crisp or fuzzy ones. The crisp criteria evaluations are assessed by means of crisp numerical values. The fuzzy criteria are presented as fuzzy preference relations or as trapezoidal fuzzy numbers. The weights of the criteria can be crisp or fuzzy as well, i.e. crisp numerical values, weighting functions or a fuzzy weighting relation between the couples of criteria according to their importance. The suggested algorithms solve the problems of choice or ordering of the alternatives from the "best" to the "worst" according to the given criteria evaluations, the weights of the criteria and the chosen algorithm.

2. Brief description of the algorithms. Let

$$A = \{a_1, \ldots, a_i, \ldots, a_n\}$$

be a set of alternatives, evaluated by the criteria

$$K = \{k_1, \ldots, k_j \ldots, k_m\}$$

and the criteria weights be

$$W = \{w_1, \ldots, w_j, \ldots, w_m\}.$$

2.1. Algorithms by crisp criteria with real numbers as weighted coefficients (ATOKRI). The set of alternatives in these algorithms are evaluated by several crisp criteria in different scales. The input data are given in matrix form

$$X = [x_{ij}], \quad i = 1, \ldots, n, \quad j = 1, \ldots, m,$$

where a real number x_{ij} is the evaluation of the alternative a_i by criterion k_j. The weighted coefficients, i.e. the importance of the criteria k_j, $j = 1, \ldots, m$ are real numbers w_j, $j = 1, \ldots, m$, as well. The aim of these algorithms is to obtain an ordering of alternatives from the best to the worst one on the basis of these data.

2.1.1. Algorithm uniting the estimations of the alternatives by aggregation operators with weighted coefficients (ATOKRI1). As the evaluations by the criteria can be in different measure units then a uniform procedure is requited. That is why, each column of the matrix X is transformed into a fuzzy preference relation using the following transformation function, e.g. the following fuzzy preference degrees are obtained from the k-th column:

$$\mu_k(a_i, a_j) = \begin{cases} 1 & \text{if} \quad i = j \\ 0.5 + \dfrac{x_{ik} - x_{jk}}{2\left(\max\limits_i\{x_{ik}\} - \min\limits_i\{x_{ik}\}\right)} & \text{if} \quad i \neq j \end{cases},$$

where $\mu_k(a_i, a_j)$, a_i, $a_j \in A$, $k \in \{k_1, k_2, \ldots, k_m\}$ is the fuzzy preference degree of the alternative a_i to a_j by the criterion k. The corresponding fuzzy relations R_k, $k = 1, \ldots, m$ are obtained from each column of matrix X, i.e. a corresponding fuzzy relation for the k-th criterion is:

$$R_k = [\mu_k(a_i, a_j)], \quad i, j = 1, \ldots, n, \ k = 1, \ldots, m. \tag{1}$$

Since some criteria can be minimized then the complements of corresponding relations to these criteria are computed to be all criteria maximized, i.e. the values of the elements of these relations are subtracted from one.

All relations R_k, $k = 1, \ldots, m$ are fused (aggregated) to obtain an aggregated fuzzy relation R with the following matrix:

$$R = [\mu(a_i, a_j)], \quad i, j = 1, \ldots, n. \tag{2}$$

Every element of this matrix is computed by means of aggregation operators with weighted coefficients $w_1, \ldots, w_j, \ldots, w_m$. The following operators are used with values $\mu_k(a_i, a_j)$ from (1):

$$\mu(a_i, a_j) = \sum_{k=1}^{m} w_k \mu_k(a_i, a_j), \quad \text{where} \ \ 0 \le w_k \le 1, \ \sum_{k=1}^{m} w_k = 1 \ [13]. \tag{3}$$

$$\mu(a_i, a_j) = \prod_{k=1}^{m} [\mu_k(a_i, a_j)]^{w_k}, \quad \text{where} \ \ 0 < w_{ki} \le 1, \ \sum_{k=1}^{m} w_k = 1 \ [14]. \tag{4}$$

The weighted coefficients for these two operators are normalized, i.e. they are transformed to values in the interval [0,1] and their sum is equal to one.

$$\mu(a_i, a_j) = \max_k \{\min(\mu_k(a_i, a_j), w_k)\}, \ \text{where} \tag{5}$$

$$0 \le w_k \le 1, \max_k \{w_k\} \ = \ 1, k = 1, \ldots, m \ \ [15].$$

$$\mu(a_i, a_j) = \ \min_k \{\max(\mu_k(a_i, a_j), 1 - w_k)\}, \quad \text{where} \tag{6}$$

$$0 \le w_k \le 1, \max_k \{w_k\} = 1, \quad k = 1, \ldots, m \ \ [15].$$

The weighted coefficients for these two operators are normalized as well, i.e. they are transformed to values in the interval [0,1] and the maximal value of them is equal to one.

Four aggregated relations are obtained, i.e. four matrices of the kind $R(2)$. These matrices are transformed to matrices R' as follows [16]:

$$\text{if } \mu(a_i, a_j) \geq \mu(a_j, a_i), \tag{7}$$

then $\mu'(a_i, a_j) = \mu(a_i, a_j)$ and $\mu'(a_j, a_i) = 0$.

The obtained matrices R' are preordered into triangular ones that present the orderings of the alternatives from the best to the worst according to the chosen aggregation operator. The algorithm is based on the investigations, published in [7, 9].

2.1.2. Algorithm uniting the estimations of the alternatives by aggregation operators without weighted coefficients (ATOKRI2). The difference with ATOKRI1 consists in the choice of the aggregation operators. Here operators are used in the mathematical expression of which the weighted coefficients are not presented. If the weights of the criteria are not given, then the first steps of the algorithm ATOKRI1 is used to obtain the fuzzy relations R_k, $k = 1, \ldots, m$ (1) and to transform the relations corresponding to the minimized criteria. The aggregated relations are computed with the help of the following operators in this case [13]:

$$\mu(a_i, a_j) = \alpha \max_k \{\mu_k(a_i, a_j)\} + (1 - \alpha) \min_k \{\mu_k(a_i, a_j)\}, \; . \\ \alpha \in [0, 1], \quad i, j = 1, \ldots, n, \quad k = 1, \ldots, m \tag{8}$$

$$\mu(a_i, a_j) = \frac{\lambda}{m} \sum_{k=1}^{m} \mu_k(a_i, a_j) + (1 - \lambda) \min_k \{\mu_k(a_i, a_j)\}, \\ \lambda \in [0, 1], \quad i, j = 1, \ldots n. \tag{9}$$

(10)

$$\mu(a_i, a_j) = \begin{cases} \left[\prod_{k=1}^{m} \mu_k(a_i, a_j)\right]^{1-\gamma} \left[1 - \prod_{k=1}^{m} (1 - \mu_k(a_i, a_j))\right]^{\gamma} & \text{if } \mu_k(a_i, a_j) \neq 0 \\ 0 & \text{otherwise} \end{cases} .$$

The values for $\mu_k(a_i, a_j)$ are taken from (1) and the coefficients α, λ, γ are assigned in addition. It can be experimented with their different values.

Three aggregated relations are obtained, i.e. three matrices of the kind $R(2)$. After that, the rest steps of the algorithm are followed, i.e. each of these matrices are transformed to a matrix R' taking into account (7). The obtained matrices R' are preordered to triangular ones, which show the ranks of the alternatives from the best to the worst one according to the chosen aggregation operator.

The steps of the algorithm are different if the weighted coefficients are given. The elements of the matrices R_k, $k = 1, \ldots, m$ (1) are transformed by the expressions [17]:

$$\mu_k^1(a_i, a_j)) = (1 - w_k) + \mu_k(a_i, a_j) - (1 - w_k)\mu_k(a_i, a_j), \quad i, j = 1, \ldots, n \tag{11}$$

and

$$\mu_k^2(a_i, a_j) = w_k \mu_k(a_i, a_j)), \tag{12}$$

where $\mu_k(a_i, a_j)$ is a corresponding element of the matrix (1). Two matrices R_k^1, R_k^2, $k = 1, \ldots, m$ are obtained for every relation R_k, $k = 1, \ldots, m$:

$$R_k^1 = \left[\mu_k^1(a_i, a_j)\right], \quad R_k^2 = \left[\mu_k^2(a_i, a_j)\right] \quad i, j = 1, \ldots, n.$$

The aggregation operators (8), (9), (10) are used to obtain the aggregated relations taking into account the new matrices:

$$\mu(a_i, a_j) = \alpha \max_k \{\mu_k^2(a_i, a_j)\} + (1 - \alpha) \min_k \{\mu_k^1(a_i, a_j)\}, \quad \alpha \in [0, 1], \quad k = 1, \ldots, m.$$

$$\mu(a_i, a_j) = \frac{\lambda}{m} \sum_{k=1}^{m} \mu_k^2(a_i, a_j) + (1 - \lambda) \min_k \{\mu_k^1(a_i, a_j)\}, \quad \lambda \in [0, 1], \quad k = 1, \ldots, m.$$

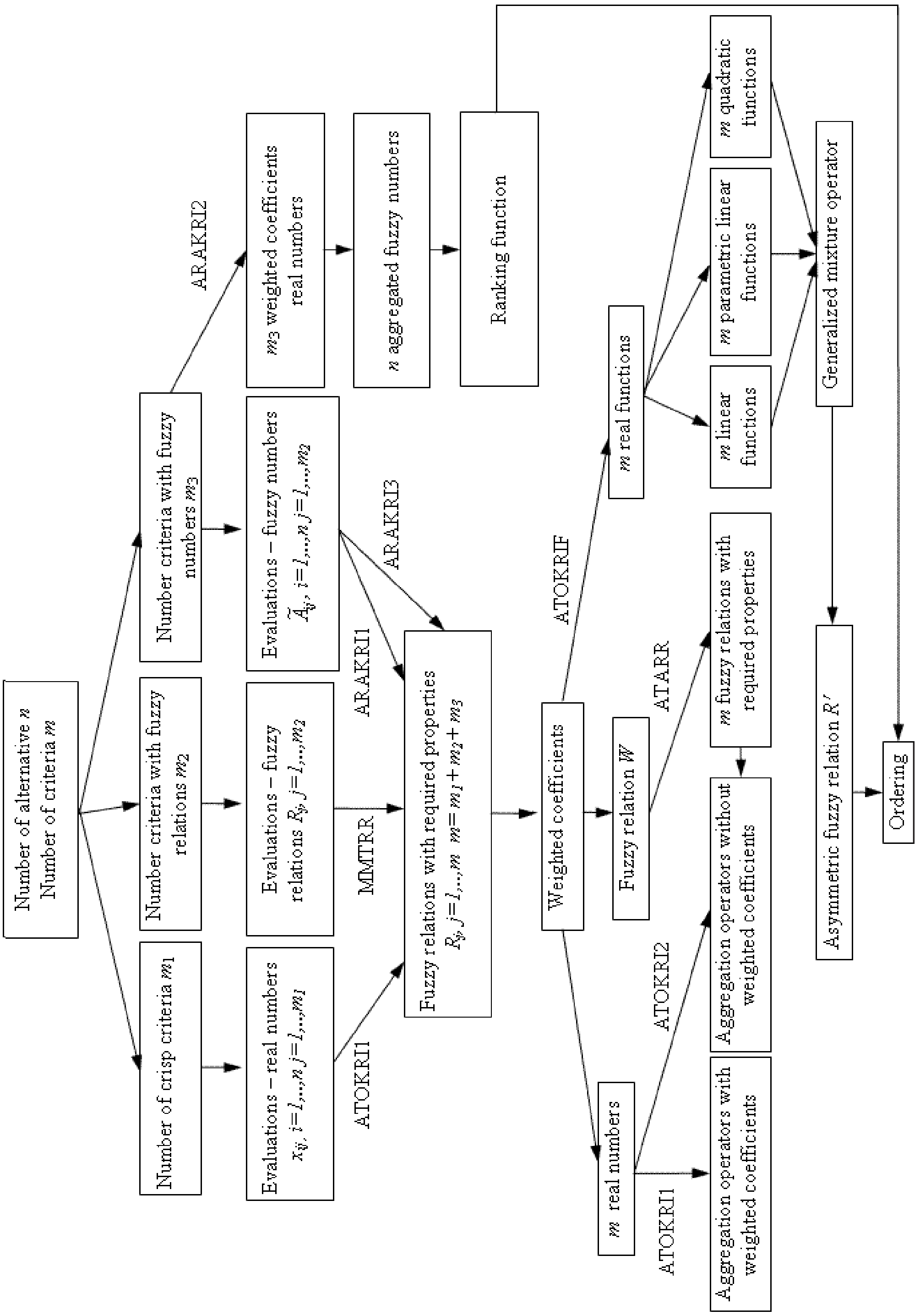
$$\mu(a_i, a_j) = \begin{cases} \left[\prod_{k=1}^{m} \mu_k^1(a_i, a_j)\right]^{1-\gamma} \left[1 - \prod_{k=1}^{m} (1 - \mu_k^2(a_i, a_j))\right]^{\gamma} & \text{if } \mu_k(a_i, a_j) \neq 0 \\ 0 & \text{otherwise} \end{cases}.$$

Three aggregated relations are obtained, i.e. three matrices of the kind $R(2)$ and the algorithm continues following the rest steps of ATOKRI1.

This algorithm is based on the investigations, published in [10].

3. Algorithm by crisp criteria with weighted coefficients – weighting functions (ATOKRIF). The difference with the algorithms ATOKRI consists in the choice of the weighted coefficients of the criteria. The suggested weighted coefficients are weighting functions [18] in this case, i.e. $f_1(x), \ldots, f_m(x)$, $x \in [0, 1]$ with arguments the elements of the corresponding matrices (1), e.g. the arguments x of the function $f_k(x)$ are the elements of the matrix R_k.

The input data of this algorithm are fuzzy relations. If the relations are obtained with the help of the algorithm ATOKRI1 then they possess the required properties to solve the problem of ordering of the alternatives. If the relations are given by experts then these relations have to be tested for the required properties.



Number of alternative n
Number of criteria m
Number of crisp criteria m1
Number criteria with fuzzy relations m2
Number criteria with fuzzy numbers m3
Evaluations – real numbers x_ij, i=1,...,n j=1,...,m1
Evaluations – fuzzy relations R_ij, j=1,...,m2
Evaluations – fuzzy numbers Ã_ij, i=1,...,n j=1,...,m2
m3 weighted coefficients real numbers
n aggregated fuzzy numbers
Ranking function
ARAKRI2
ATOKRI1
MMTRR
ARAKRI1
ARAKRI3
Fuzzy relations with required properties R_ij, j=1,..,m m=m1+m2+m3
Weighted coefficients
ATOKRIF
m real functions
m linear functions
m parametric linear functions
m quadratic functions
Generalized mixture operator
Fuzzy relation W
ATARR
m fuzzy relations with required properties
m real numbers
ATOKRI1
ATOKRI2
Aggregation operators with weighted coefficients
Aggregation operators without weighted coefficients
Asymmetric fuzzy relation R′
Ordering

Fig. 1

If they do not possess the defined properties then the corresponding matrices are transformed into matrices of the fuzzy relations with required properties. The algorithm MMTRR considered below is used for this purpose. Let the obtained relations by the algorithm ATOKRI1 or the given relations by experts be $R_k,\ k = 1, \ldots, m$. The weighting functions are assigned with the help of corresponding parameters possessing required properties [18]:

Linear functions:

$$f_k(x) = 1 + \beta_k x \quad \text{with parameters} \quad 0 \le \beta_k \le 1, \quad k = 1, \ldots, m, \quad m \ge 2; \tag{13}$$

Parametric linear functions:

$$f_k(x) = \alpha_k \frac{1 + \beta_k x}{1 + \beta_k} = \gamma_k(1 + \beta_k x), \tag{14}$$

where

$$0 < \alpha_k \le 1, \quad 0 \le \beta_k \le 1, \quad \gamma_k = \frac{\alpha_k}{1 + \beta_k}, \quad k = 1, \ldots, m;$$

Quadratic functions:

$$f_k(x) = 1 + (\beta_k - \gamma_k)x + \gamma_k x^2, \tag{15}$$

with parameters

$$\beta_k \ge 0, \quad \gamma_k \ge 0, \quad k = 1, \ldots, m.$$

The functions' parameters are different for the separate functions and they have not a relationship between them.

New membership degrees are computed for each relation R_k, $k = 1, \ldots, m$ taking into account the corresponding weighting function as follows [18]:

$$\mu_k^w(a_i, a_j) = \begin{cases} 1 & \text{if} \quad a_i = a_j \\ \dfrac{f_k(\mu_k(a_i, a_j))\mu_k(a_i, a_j)}{S(a_i, a_j)} & \text{if} \quad a_i \ne a_j \end{cases} \qquad i, j = 1, 2, 3, \quad k = 1, 2,$$

where $S(a_i, a_j) = \sum_{k=1}^{m} f_k(\mu_k(a_i, a_j))$, and $f_k(\bullet)$ is a weighting function from (13), (14), (15). Therefore, the following matrices $R_k^w = [\mu_k^w(a_i, a_j)]$, $k = 1, \ldots, m$ are obtained.

These matrices are aggregated into one matrix R (2) which elements are computed summing up the corresponding elements of the matrices R_k^w, $k = 1, \ldots, m$. On the base of this matrix R, the rest steps of the algorithm ATOKRI1 follow. Therefore, R is transformed into a matrix R' taking into account (7). The obtained matrix R' is preordered into a triangular matrix, which shows the rank ordering of the alternatives according to the chosen weighting function.

The algorithm is based on the investigations, published in [12].

4. Algorithm for aggregation of fuzzy relations between alternatives and a fuzzy relation between the weights (importance) of the criteria (ARAKRI). The evaluations of the criteria can be: as crisp data of the matrix $X = [x_{ij}]$, $i = 1, \dots, n$, $j = 1, \dots, m$ and/or as fuzzy relations (1). The fuzzy relations between the couples of alternatives for the crisp criteria are computed by the first steps of the algorithm ATOKRI1. If the criteria evaluations are fuzzy relations then they are tested for required properties and after that they are transformed into the relations with the required properties using the algorithm ATARR, considered below. The weighted coefficients are given as a fuzzy relation with defined properties:

$$(16) \qquad W = [w_{ij}], \quad w_{ii} = 0.5, \quad w_{ij} = 1 - w_{ji}, \quad i, j = 1, \dots, m.$$

The aggregation of every couple of matrices (relations) R_k, $k = 1, \dots, m$, including the corresponding elements of the matrix W (16) are computed with the help of [19] and a new aggregation operator. For example, this new operator fuses R_i and R_j taking into account the corresponding elements of the matrix W. A new relation R_{ij} is obtained and according to this operator $R_{ij} = R_{ji}$. The number p of the new relations is equal to the composition of two elements from m (number of criteria), i.e. $p = \dfrac{m(m-1)}{1.2}$. The algorithm ATOKRI2 for aggregation operators without weighted coefficients are used for fusion of these p new relations. The rest steps of ATOKRI2 are followed.

This algorithm is based on the investigations, published in [11, 12].

5. Algorithms for fuzzy numbers as alternatives' evaluations. ***5.1. Algorithms for evaluations – fuzzy numbers by all criteria.*** The input data for these algorithms have to be fuzzy or real numbers by all criteria for all alternatives (real numbers are a private case of the fuzzy numbers). Two approaches suggested to solve the problem of the decision making. The first approach uses a ranking function [24] to transform the fuzzy number into a real index (ARAKRI1). The aggregation of these indices is computed by aggregation operators in according with the given weighted coefficients of the criteria. The second approach (ARAKRI2) aggregates the fuzzy numbers directly with the help of aggregation operators.

5.1.1. Algorithm with a ranking function (ARAKRI1). The evaluations of the alternatives by the criteria are fuzzy numbers written in matrix firm:

$$(17) \qquad \tilde{A} = \left[\tilde{A}_{ij}\right] \quad i = 1, \dots, n, \quad j = 1, \dots, m,$$

where the fuzzy number $\tilde{A}_{ij} = (a_{ij}^1, a_{ij}^2, a_{ij}^3, a_{ij}^4)$ is the evaluation of the i-th alternative by the j-th criterion and $a_{ij}^1 \le a_{ij}^2 \le a_{ij}^3 \le a_{ij}^4$ are real numbers. The fuzzy numbers can be from different scales for the separate criteria.

If the evaluations are from different scales and they are not in the interval [0,1], then a procedure for an unification and a normalization of the fuzzy numbers is performed. This procedure does not change the order of the fuzzy numbers.

Let [20] $a_j^{\max} = \max_i \{a_{ij}^4\}$, $a_j^{\min} = \min_i \{a_{ij}^1\}$, $da = a_j^{\max} - a_j^{\min}$, then the unified and normalized fuzzy numbers $\tilde{Z}_{ij} = (z_{ij}^1, z_{ij}^2, z_{ij}^3, z_{ij}^4)$, $i = 1, \ldots, n$, $j = 1, \ldots, m$ is computed by the expression:

$$\tilde{Z}_{ij} = (\tilde{A}_{ij} - a_j^{\min})/da, \quad i = 1, \ldots, n, \quad j = 1, \ldots, m.$$

If one part of the criteria is maximized, and the other part is minimized, then the complements of the fuzzy numbers to the fuzzy number (1,1,1,1) for these minimized criteria are computed to be the criteria from the same kind. Then, the indices $F(\tilde{Z}_{ij})$ of the unified and normalized fuzzy numbers are computed using the following ranking function:

$$F(\tilde{Z}_{ij}) = kF_1(\tilde{Z}_{ij}) + (1-k)F_2(\tilde{Z}_{ij}), \quad k \in [0,1], \quad \text{where} \tag{18}$$

$$F_1(\tilde{Z}_{ij}) = a_{ij}^1 + \frac{(a_{ij}^4 - a_{ij}^1) + (a_{ij}^3 - a_{ij}^2)}{2} \times \frac{1}{\sqrt{(a_{ij}^4 - a_{ij}^3)^2 + 1}},$$

$$F_2(\tilde{Z}_{ij}) = a_{ij}^4 - \frac{(a_{ij}^4 - a_{ij}^1) + (a_{ij}^3 - a_{ij}^2)}{2} \times \frac{1}{\sqrt{(a_{ij}^2 - a_{ij}^1)^2 + 1}}.$$

Therefore, the transformed evaluations are already real numbers and the classical problem for decision making is obtained.

• If the criteria have equal weights then the aggregation operators without weighted coefficients are used, i.e. the algorithm ATOKRI2 with matrices – columns has to be used;

• If the weights of the criteria are different real numbers, then aggregation operators with weighted coefficients are used (algorithm ATOKRI1) or aggregation operators without weighted coefficients are used but after weighted transformations of the evaluations (algorithm ATOKRI2 with matrices – columns).

• If the weights of the criteria are real functions then the algorithm ATOKRIF is used.

The algorithm is based on the investigations, published in [8, 16, 21, 22].

5.1.2. Algorithm for aggregation of fuzzy numbers directly (ARAKRI2). This algorithm uses the aggregation operators and operations between fuzzy numbers to fuse the fuzzy numbers by all criteria corresponding to the separate alternatives. The first steps are the same as the ones in the algorithm ARAKRI1. The maximal and the minimal fuzzy number from a given series of those numbers are obtained from the matrix (17) with the indices of unified and normalized fuzzy numbers.

• If the criteria have equal weights then the aggregation operators without weighted coefficients are used, i.e. the algorithm ATOKRI2;

• If the assign weights of the criteria are different real numbers, then they are normalized and the aggregation operators with weighted coefficients are used (algorithm ATOKRI1) or aggregation operators without weighted coefficients are used but the fuzzy numbers are multiple with the corresponding weighted coefficients (algorithm ATOKRI2).

Aggregated fuzzy numbers corresponding to the alternatives are obtained as a result of these computations. After that, these fuzzy numbers have to be ordered by greatness. The indices (18) of these aggregated evaluations are computed for this purpose. Then, the order of alternatives corresponds to the ordering of the obtained indices.

The algorithm is based on the investigations, published in [6, 8, 23].

5.2. Algorithm for mixed information (ARAKRI3). Let the set of alternatives be evaluated by different criteria, e.g. crisp evaluations, fuzzy relations, fuzzy numbers. This heterogeneous information has to be reduced to one measureless scale. A based approach to uniform the information is to obtain fuzzy relations from the given data comparing the evaluations of the couples of alternatives for every criterion. The algorithm ARAKRI3 computes the fuzzy relations for the criteria which evaluate alternatives by fuzzy numbers. Every couple of fuzzy numbers can be compared using the index (18) to obtain the membership degree of the given couple to a fuzzy relation. The algorithm ARAKRI1 is used to compute the indices (18) of the fuzzy numbers from the matrix with unified and normalized fuzzy numbers. After that, the following value is computed for every couple a_i, a_j of alternatives for every criterion (e.g. for the k-th criterion):

$$\mu_k(a_i, a_j) = 0.5 + \frac{F(\tilde{Z}_{ik}) - F(\tilde{Z}_{jk})}{2(F_k^{\max} - F_k^{\min})},$$

where $F_k^{\max}$, $F_k^{\min}$ are the indices of the greatest and the smallest fuzzy number for the set of fuzzy numbers corresponding to the k-th criterion.

In this way, a fuzzy relation (1) is obtained corresponding to every criterion with fuzzy numbers as the alternatives' evaluations. The so obtained fuzzy relations are used for calculations in the algorithms for criteria with mixed information ATOKRI.

The algorithm is based on the investigations, published in [6].

6. Algorithms for testing and obtaining of the fuzzy relations with defined properties. *6.1. Algorithm MMTRR for max-min transitive fuzzy relations.*

A) The algorithm tests a fuzzy relation for the max-min transitive property, i.e. if the fuzzy relation is $R = \|r_{ij}\|$, $r_{ii} = 1$, $i, j = 1, \ldots, n$ whether $r_{ij} \geq \min(r_{ik}, r_{kj})\}$, $\forall i, j, k = 1, \ldots, n$.

B) The algorithm transforms a given relation R to a max-min transitive relation. It uses the definition of the transitive closure of a fuzzy relation R , i.e. the transitive fuzzy relation R_T, that contains R and $R_T = R \cup R^2 \cup \cdots \cup R^n$.

6.2. Algorithm ATARR for additive transitive reciprocal relations.

A) The algorithm tests a fuzzy reciprocal relation for additively transitive property, i.e. if $R = \|r_{ij}\|$, $r_{ii} = 0.5$, $r_{ij} = 1 - r_{ji}$, $i, j = 1, \ldots, n$, then whether $r_{ij} + r_{jk} + r_{ik} = 1.5$ $\forall i, j, k = 1, \ldots, n$.

B) The algorithm constructs an additively transitive relation from a given reciprocal relation. An algorithm given in [3] is used.

7. Arrangement of the programme package containing the FMCDM algorithms. The **input data** for the package are:

- n = a number of alternatives;
- m = a number of criteria;
- numbers of criteria with homogeneous evaluations, for example m_1 = a number of crisp criteria, m_2 = a number of criteria, that evaluate the alternatives with fuzzy relations, m_3 = a number of criteria with evaluations fuzzy numbers, where

$$m_1 + m_2 + m_3 = m, \quad m_1, m_2, m_3 = 0, \ldots, m.$$

- $m_1 = m$ means that all criteria are crisp;
- $m_2 = m$ means that all criteria assign fuzzy relations;
- $m_3 = m$ means that all criteria assign fuzzy numbers for alternatives' evaluations;

  — evaluations of the alternatives by the criteria as follows:
- the evaluations for the crisp criteria have to be written as the elements of the matrix

$$X = [x_{ij}] \ , \ i = 1, \ldots, n, \ j = 1, \ldots, m;$$

- for the fuzzy relations have to be written in the matrix form (1);
- for the fuzzy numbers have to be written as (17);

  — weighted coefficients of the criteria can be:
- real numbers $w_1, \ldots, w_j, \ldots, w_m$;
- parameters of the real functions $f_1(x), \ldots, f_m(x)$, $x \in [0, 1]$, where $f_j(x)$, $j = 1, \ldots, m$ are functions of the kind (13), (14), (15);
- a matrix of the fuzzy preference relation W (16) between the criteria' importance.

It is possible to use the following combinations of input data:

A) All criteria are crisp, i.e. $m_1 = m$ and the weighted coefficients are real numbers $w_1, \ldots, w_m$ – the crisp evaluations by the criteria are transformed into fuzzy preference relations with defined properties and the computations are completed with the help of the algorithm ATOKRI1 or the algorithm ATOKRI2, where weighted transformations of the obtained fuzzy relations by ATOKRI1 are performed and after that aggregation operators without weighted coefficients are used;

B) All criteria are crisp, i.e. $m_1 = m$ and the weighted coefficients of the criteria are equal, i.e. the criteria are without weights – the crisp evaluations by the criteria are transformed into fuzzy preference relations with defined properties and the computations are completed with the help of the algorithm ATOKRI2;

C) All criteria are crisp, i.e. $m_1 = m$ and the weighted coefficients of the criteria are real functions $f_1(x), \ldots, f_m(x),\ x \in [0, 1]$ – the crisp evaluations by the criteria are transformed into fuzzy preference relations with defined properties and the computations are completed with the help of the algorithm ATOKRI1, after that the algorithm ATOKRIF is used;

D) All criteria are crisp, i.e. $m = m_1$ and a fuzzy preference relation between the criteria' importance W is given – the crisp evaluations by the criteria are transformed into fuzzy preference relations with defined properties and the computations are completed with the help of the algorithm ATOKRI1, after that the algorithm ARAKRI is used;

E) All criteria give fuzzy preference relations between the couples of alternatives, i.e. $m = m_2$ and the weighted coefficients are real numbers $w_1, \ldots, w_m$ – the fuzzy relations are tested for defined properties by the algorithm MMTRR and those of them that do not possess the required properties are transformed into new relations holding these properties. The aggregation operators with weighted coefficients given in the algorithm ATOKRI1 are used and/or the algorithm ATOKRI2 is used, where weighted transformations of the obtained by MMTRR fuzzy relations are performed. Aggregation operators without weighted coefficients are used, after that;

F) All criteria give fuzzy preference relations between the couples of alternatives, i.e. $m = m_2$ and the weighted coefficients are real functions $f_1(x), \ldots, f_m(x)$, $x \in [0, 1]$ (all of them have to be linear, or parametric linear, or quadratic functions) – the fuzzy relations are tested for defined properties by the algorithm MMTRR and those of them that do not possess the required properties are transformed into new relations holding these properties. The algorithm ATOKRIF is used, after that;

G) All criteria give fuzzy preference relations between the couples of alternatives, i.e. $m = m_2$ and a fuzzy preference relation between criteria' importance W is given, as well – the fuzzy relations are tested for defined properties by the algorithm ATARR and those of them that do not possess the required properties are transformed into new relations holding these properties. The algorithm ARAKRI is used, after that;

H) All criteria assign fuzzy numbers as evaluations of the alternatives, i.e. $m = m_3$ and the weighted coefficients are real numbers $w_1, \ldots, w_m$ – the algorithms ARAKRI1 and/or ARAKRI2 can be used;

I) All criteria assign fuzzy numbers as evaluations of the alternatives, i.e. $m =$

m_3 and the weighted coefficients are real functions $f_1(x), \ldots, f_m(x),\ x \in [0,1]$ (all of them have to be linear, or parametric linear, or quadratic functions) – the algorithms ARAKRI1 and/or ARAKRI2 can be used. The algorithm ATOKRIF is required, after that;

J) The criteria are different, i.e. $m = m_1+m_2+m_3$. The algorithm ATOKRI1 is used for the criteria with the number m_1, the algorithm MMTRR is required for the criteria with the number m_2 and the algorithm ARAKRI3 is used for the criteria with the number m_3. Since all algorithms transformed the initial information into fuzzy preference relations then the following computations are reduced to the cases E), F), G) according to the given weighted coefficients.

The output data for the package are fuzzy preference linear orderings of the alternatives from the best to the worst one according to the chosen approach, the aggregation operator or the given weighted coefficients. These linear orderings are fuzzy, because they show the preference degrees of one alternative before other, too.

8. Concluding remarks. The suggested programme package works into the Excel area and it contains the traditional algorithms for solving FMCDM problems as well as the author's algorithms. Different input data may be used and different ordering may be obtained according to the chosen algorithms, weighted coefficients and parameters of the aggregation operators. The package is examined for the purpose of projects [1] directed towards researching and developing of the models in multi-criteria Decision Support Systems (DSS).

The diffusion of the fuzzy sets theory into the MCDM methods and the review of the published papers show that the programme package of such kind do not perform, as may be seen in [1, 2, 4, 5]. The recent developments and algorithms in the area of FMCDM are considered in [2]. The last results of the investigations connected with the MCDM problems are given in [1, 4]. One of the most significant events [5] in this area, the International Conference on Multiple Criteria Decision Making, shows the original research results and the practical development experiences among FMCDM. The conference seeks solutions to challenging problems facing the development of MCDM, and shapes future directions of research by promoting high quality, novel and daring research findings, important contributions to MCDM and/or creative thinking towards the future development of MCDM.

REFERENCES

[1] Angelova V. Cybernetics and Information Technologies. Bulgarian Academy of Sciences, **9**, 2009, No 1, 18–24.

[2] Carlsson Ch., R. Fuller. Fuzzy Sets and Systems, **78**, 1996, No 2, 139–153.

[3] Herrera-Viedma E., F. Herrera, F. Chiclana, M. Luque. European Journal of Operational Research, **154**, 2004, No 1, 98–109.
[4] Kaharaman C. (ed.) Fuzzy multi-criteria decision making. Theory and applications with recent developments. Springer Optimization and its applications, **16**, 2008, Springer Science + Business Media, LLC.
[5] MCDM 2009, the 20th Intern. Conf. on Multiple Criteria Decision Making, Chengdu/Jiuzhaigou, China, June 21–26, 2009, `http://www.mcdm2009.cn`
[6] Peneva V., I. Popchev. Fuzzy Sets and Systems, **97**, 1998, No 1, 75–81.
[7] Peneva V., I. Popchev. Cybernetics and Systems, **30**, 1999, No 8, 725–745.
[8] Peneva V., I. Popchev. Cybernetics and Systems, **32**, 2001, No 8, 821–835.
[9] Peneva V., I. Popchev. Fuzzy Sets and Systems, **139**, 2003, No 3, 615–633.
[10] Peneva V., I. Popchev. Advanced Studies in Contemporary Mathematics, **10**, 2005, No 1, 25–44.
[11] Peneva V., I. Popchev. Fuzzy Optimization and Decision Making, **6**, 2007, No 4, 351–365.
[12] Peneva V., I. Popchev. Advanced Studies in Contemporary Mathematics, **15**, 2007, No 1, 121–132.
[13] Von Altrock C. Fuzzy Logic and Neuro Fuzzy Applications in Business and Finance. Englewood Cliffs, NJ: Prentice Hall, 1997.
[14] Chiclana F., F. Herrera, E. Herrera-Viedma. Proc. and Manag. of Univ. In: Knowledge-Bases Systems, IPMU'2000, **II**, DECSAI University of Granada, 2000, 985–991.
[15] Fodor J. C., M. Roubens. Inform. Sci, **84**, 1995, No 3–4, 173–180.
[16] Nakamura K. Fuzzy Sets and Systems, **20**, 1986, No 2, 147–162.
[17] Yager R. R. Uncertainty, Fuzziness and Knowledge-based Systems, **2**, 1994, No 1, 101–114.
[18] Ribeiro R. A., R. A. Marques Pereira. European Journal of Operational Research, **145**, 2003, No 2, 329–342.
[19] Nguyen H. T., E. A. Walker. A First Course in Fuzzy Logic (second edition). Chapman & Hall/CRC, 2000.
[20] Heilpern St. Cybernetics and Systems'94, (ed. R. Trappl), **II**, Singapore, World Scientific, 1994, 279–285.
[21] Czyzak P., R. Slovinski. Proc. of Xth Intern. Conf. on MCDM, Coimbra, Portugal. August 1994, Multicriteria Analysis. Springer, N.Y., 1994, 85–93.
[22] Yuan Y. Fuzzy Sets and Systems, **43**, 1991, No 2, 139–157.
[23] Buckley J. J. Fuzzy Sets and Systems, **33**, 1989, No 1, 119–121.
[24] Kim K., K. S. Park. Fuzzy Sets and Systems, **35**, 1990, No 2, 143–150.

Institute of Information Technologies
Bulgarian Academy of Sciences
Acad. G. Bonchev Str., Bl. 2
1113 Sofia, Bulgaria