Доклади на Българската академия на науките
Comptes rendus de l'Académie bulgare des Sciences
Tome 60, No 9, 2007

SCIENCES ET INGENIERIE
Théorie des systèmes

A RELAXED SUFFICIENT CONDITION FOR ROBUST STABILITY OF AFFINE SYSTEMS

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(*Submitted on June 25, 2007*)

Abstract

A new sufficient condition for robust stability of linear time-invariant systems is proposed. It is based on the proved equivalence between stability of an augmented system and stability of the original one. This helps to solve the problem for the first system by making use of certain additional matrix variables. The suggested approach is less conservative than the existing similar ones which represent particular cases of the proposed here sufficient robust stability condition.

Key words: uncertain systems, robust stability, polynomial Lyapunov functions

1. Introduction. Robustness analysis of linear time-invariant systems subjected to parameter uncertainty, belonging to some vector set, attracts the attention of the control theory community for more than three decades. In particular, the LMI-based approach has become very promising, since it admits the usage of parameter-dependent Lyapunov functions. As a result, less conservative conditions for robust stability can be derived (see, e.g., [1, 2]). An LMI approach for robust stability analysis via Lyapunov functions associated with higher-order state vector derivatives was recently proposed in [3]. The objective of this paper is to generalize this approach. For the purpose, the state-space description of an augmented system is introduced. It is proved that the stability of this system is equivalent to the stability of the original one. The main advantage consists in the possibility to use some additional arbitrary parameter-dependent matrix variables. It is shown that the LMI problem can also be convexified. Thus, the problem solution consists in a check for negative definiteness of a finite number of matrices. The main result is a sufficient condition for robust stability which is proved to be less conservative than the ones proposed by similar approaches.

2. Preliminaries. The following notations are used throughout the paper. $\mathbf{R}_{r,q}$, R_r, H_r denote the sets of real $r \times q$, $r \times r$ and $r \times r$ Hurwitz (negative stable) matrices, respectively, and $\mathbf{R}$ is the set $r \times 1$ real vectors. An $r \times q$ zero block and the $r \times r$ identity matrix are denoted $0_{r,q}$, I_r, respectively. The set of eigenvalues (spectrum) of a square matrix X is $\sigma(X)$. The set of $p \times n$ real matrices of rank $n \leq p$, is denoted $Z(p,n)$. Some basic for the present research results are given below.

This work is supported by the Bulgarian Academy of Sciences under grant 010077/2007.

Theorem 2.1. The following statements are equivalent:

(a) $A \in \mathbf{H}_n$;

(b) for any given matrices $\tilde{B} \in \mathbf{Z}(p,n)$ and $X \in \mathbf{H}_{p-n}$, there exist matrix $\tilde{A} \in \mathbf{H}_p$ and nonsingular $S \in \mathbf{R}_p$, such that $\tilde{A}B = BA$, $\sigma(\tilde{A}) \equiv \sigma(A) \cup \sigma(X)$, $B = S\tilde{B} \in Z(p,n)$. Matrix $\tilde{A}$ is similar to an upper block triangular matrix, with the off-diagonal block being arbitrarily chosen;

(c) there exists a positive definite matrix $\Pi \in \mathbf{R}_p$, such that for any given $B \in \mathbf{Z}(p,n)$, one has $L = A^T P + PA < 0$, $P = B^T \Pi B$.

Proof. Let (a) holds and $B \in \mathbf{Z}(p,n)$, $R \in \mathbf{R}_{n,p-n}$ and $G \in \mathbf{R}_{p-n}$ are arbitrary matrices. There exist some $p \times p$ nonsingular matrices S and T, such that

(1)
$$S\tilde{B} = \begin{bmatrix} I_n \\ C \end{bmatrix} = B, \qquad T = \begin{bmatrix} I_n & 0_{n,p-n} \\ C & I_{p-n} \end{bmatrix},$$
$$T\tilde{A}T^{-1} = \begin{bmatrix} A & R \\ 0_{p-n,n} & -(CR+G) \end{bmatrix}, \quad \tilde{A} = \begin{bmatrix} A - RC & R \\ CA + GC & -G \end{bmatrix}.$$

Then, $\tilde{A}B = BA$, $B \in \mathbf{Z}(p,n)$. Since R and G are arbitrary matrices, for $G = -CR - X$, one gets $\sigma(\tilde{A}) \equiv \sigma(T\tilde{A}T^{-1}) \equiv \sigma(A) \cup \sigma(X)$, i.e., $\tilde{A} \in \mathbf{H}_p$. This proves (a) $\Rightarrow$ (b).

Let (b) holds. There exists some positive definite matrix $\Pi \in \mathbf{R}_p$, such that $\tilde{L} = \tilde{A}^T \Pi + \Pi \tilde{A} < 0$. For any $B \in \mathbf{Z}(p,n)$, one has $0 > B^T \tilde{L} B = A^T B^T \Pi B + B^T \Pi B A = L$, which proves (b) $\Rightarrow$ (c).

If (c) holds, then obviously $A \in \mathbf{H}_n$, i.e., (c) $\Rightarrow$ (a).

Corollary 2.1. $A \in \mathbf{H}_n$ if and only if for any given matrices $B \in \mathbf{Z}(p,n)$ and $F \in \mathbf{Z}(m,p)$, there exists a positive definite matrix $\Pi \in \mathbf{R}_m$, such that $\tilde{A}^T F^T \Pi F + F^T \Pi F \tilde{A} < 0$, $\tilde{A}B = BA$.

Proof. It follows from Theorem 2.1.

The following result is a direct consequence from Corollary 2.1.

Corollary 2.2. Consider the dynamic systems

(2)
$$\dot{x} = Ax, \quad x \in \mathbf{R}^n;$$

(3)
$$\dot{\mu} = \tilde{A}\mu, \quad \mu = Bx \in \mathbf{R}^p, \quad B \in \mathbf{Z}(p,n), \quad \tilde{A}B = BA.$$

System (2) is asymptotically stable if and only if for any given matrices B in (3) and $F \in \mathbf{Z}(m,p)$, there exists a positive definite matrix $\Pi \in \mathbf{R}_m$, such that $v(\mu) = \mu^T F^T \Pi F \mu$ is a valid Lyapunov function (VLF) for system (3), i.e., $v(\mu) > 0$, $\dot{v}(\mu) < 0$, $\forall \mu \neq 0$. If $v(\mu)$ is a VLF for system (3), then $v(x) = x^T B^T F^T \Pi F B x$ is VLF for system (2) and vice versa.

For any integer $f \geq 1$ denote $m = pf$ and

(4)
$$F = \begin{bmatrix} I_p \\ \tilde{A} \\ \tilde{A}^2 \\ \vdots \\ \tilde{A}^{f-1} \end{bmatrix}, \quad F\mu = \begin{pmatrix} \mu \\ \mu^{(1)} \\ \mu^{(2)} \\ \vdots \\ \mu^{(f-1)} \end{pmatrix} = \mu_1, \quad \mu_2 = \begin{pmatrix} \mu_1 \\ \dot{\mu}_1 \end{pmatrix},$$
$$\tilde{\mu} = \begin{pmatrix} \mu_1 \\ \mu^{(f)} \end{pmatrix}, \quad E = \begin{bmatrix} E_1 \\ E_2 \end{bmatrix}, \qquad \bar{\Pi} = \begin{bmatrix} 0_m & \Pi \\ \Pi & 0_m \end{bmatrix},$$

where $F \in \mathbf{Z}(m,p)$, $\mu_1 \in \mathbf{R}^m$, $\mu_2 \in \mathbf{R}^{2m}$, $\tilde{\mu} \in \mathbf{R}^{m+p}$, $E_1 = [I_m \; 0_{m,p}]$, $E_2 = [0_{m,p} \; I_m]$, $E \in \mathbf{Z}(2m,\, m+p)$, $\bar{\Pi} \in \mathbf{R}_{2m}$, $\Pi \in \mathbf{R}_m$ and $\mu^{(i)}$ denotes the i-th total time derivative of μ, e.g., $\mu^{(1)} \equiv \dot{\mu}$. Note that $\mu_2 = E\tilde{\mu}$ and the total time derivative of the candidate for a Lyapunov function for system (3) is computed as follows:

$$\dot{v}(\mu) = 2\mu^T F^T \Pi F \dot{\mu} = 2\mu_1^T \Pi \dot{\mu}_1 = \mu_2^T \bar{\Pi} \mu_2 = \tilde{\mu}^T E^T \bar{\Pi} E \tilde{\mu}. \tag{5}$$

Define the following matrices: bl. diag. $[\tilde{A}] = \tilde{A}_d \in \mathbf{R}_m$, $U = [\tilde{A}_d \; -I_m] \in \mathbf{R}_{m,2m}$ and $\bar{A} = [\bar{A}_{ij}] \in \mathbf{R}_{m,m+p}$, $\bar{A}_{ij} \in \mathbf{R}_p$, $i = 1,2,\dots,f$, $j = 1,2,\dots,f+1$,

$$\bar{A}_{ij} = \begin{cases} A, & i = j \\ -I_p, & j = i+1 \\ 0_p, & \text{otherwise.} \end{cases}$$

Theorem 2.2. Consider the following $(m+p) \times (m+p)$ symmetric matrix

$$\bar{L} = E^T \bar{\Pi} E + \bar{A}^T K^T + K\bar{A}, \quad K \in \mathbf{R}_{m+p,m}. \tag{6}$$

System (2) is asymptotically stable if and only if for any given matrices B in (3) and F in (4) there exist a positive definite matrix $\Pi \in \mathbf{R}_m$ and matrix K, such that $\bar{L}$ in (6) is a negative definite matrix.

Proof. Let system (2) be asymptotically stable. For any given matrices B in (3) and F in (4) there exists some positive definite $\Pi \in \mathbf{R}_m$, such that $v(\mu) = \mu^T F^T \Pi F \mu$ is a VLF for system (3), in accordance with Corollary 2, i.e., $\tilde{A}^T \tilde{P} + \tilde{P}\tilde{A} = L < 0$, $\tilde{P} = F^T \Pi F$. Consider the matrices

$$\tilde{K} = \begin{bmatrix} K_1 \\ K_2 \end{bmatrix} \in \mathbf{R}_{2m,m},$$

$$\tilde{L} = \bar{\Pi} + U^T \tilde{K}^T + \tilde{K} U = \begin{bmatrix} \tilde{A}_d^T K_1^T + K_1 \tilde{A}_d & \Pi + \tilde{A}_d^T K_2^T - K_1 \\ \Pi + K_2 \tilde{A}_d - K_1^T & -(K_2^T + K_2) \end{bmatrix} = \begin{bmatrix} L_{11} & L_{12} \\ L_{121}^T & L_{22} \end{bmatrix} \in \mathbf{R}_{2m}.$$

Let $K_1 = \Pi + \tilde{A}_d^T K_2^T$. Then, $L_{11} = \tilde{A}_d^T \Pi + \Pi \tilde{A}_d^- \tilde{A}_d^T L_{22} \tilde{A}_d$ and $L_{12} = 0_m$. For $\Pi =$ bl.diag.$[\tilde{P}]$, one has $L_{11} =$ bl.diag.$[L] - \tilde{A}_d^T L_{22} \tilde{A}_d$, bl.diag.$[L] < 0$. There always exists some appropriate matrix K_2, such that $L_{22} < 0$ and $L_{11} < 0$, i.e., $\tilde{L} < 0$. Since $E \in \mathbf{Z}(2m,\, m+p)$ and $UE = \bar{A}$, one gets $0 > E^T \tilde{L} E = E^T \bar{\Pi} E + \bar{A}^T K^T + K\bar{A} = \bar{L}$, $K = E^T \tilde{K}$, which proves the necessity part.

Let $\bar{L}$ in (6) be a negative matrix for some matrices B in (3), F in (4), $\Pi > 0$ and K. Since $\bar{A}\tilde{\mu} = 0 \in \mathbf{R}^m$, $\mu_2 = E\tilde{\mu}$ and having in mind (5), one has

$$0 > \tilde{\mu}^T \bar{L} \tilde{\mu} = \tilde{\mu}^T (E^T \bar{\Pi} E + \bar{A}^T K^T + K\bar{A}) \tilde{\mu} = \mu_2^T \bar{\Pi} \mu_2 = \dot{v}(\mu).$$

This means that $v(\mu) = \mu^T F^T \Pi F \mu$ is a VLF for system (3) and in accordance with Corollary 2, $v(x) = x^T B^T F^T \Pi F B x$ is a VLF for system (2), i.e., it is asymptotically stable. This proves the sufficiency part.

Comments. Theorem 2.2 proves the equivalence between negative definiteness of $\bar{L}$ in (6) and the existence of matrices B in (3), F in (4) and $\Pi(m)$, $m = pf$, $p \geq n$, $f \geq 1$, such that

$$L(m) = \tilde{A}^T \tilde{\Pi}(m) + \tilde{\Pi}(m)\tilde{A} < 0; \quad \tilde{\Pi}(m) = F^T \Pi(m) F, \;\; \Pi(m) > 0, \;\; \tilde{A}^T B = BA. \tag{7}$$

For the particular case $p = n$, $m = nf$, $B = I_n$, (7) becomes

$$L(nf) = A^T\tilde{\Pi}(nf) + \tilde{\Pi}(nf)A < 0; \quad \tilde{\Pi}(nf) = F^T\Pi(nf)F, \ \ \Pi(nf) > 0, \tag{8}$$

and the condition based on negative definiteness of the corresponding matrix $\bar{L}$ in (6) was proposed for $f = 2$ in [4] and for $f \geq 2$, in [3]. Obviously, Corollary 2.2 and Theorem 2.2 generalize these results. By making use of Theorem 2.1 and Corollary 2.1, one can easily show that inequality (8) always implies inequality (7) for arbitrary $B \in \mathbf{Z}(p,n)$, $p > n$ and $F \in \mathbf{Z}(m,p)$. Therefore, the condition proposed by Theorem 2.2 does not introduce any conservatism for $p > n$, with respect to the particular case $p = n$, in the stability analysis for system (2).

3. Uncertain dynamic systems. Consider the set of constant, but not exactly known vectors $\mathbf{\Omega} = \{\alpha \in \mathbf{R}^N : \ \alpha_{\min i} \leq \alpha_i \leq \alpha_{\max i}, \ \alpha_{\min i} \leq 0 \leq \alpha_{\max i}, \ i = 1,2,\dots,N\}$ and the following uncertain dynamic systems:

$$\dot{x} = A(\alpha)x, \quad x \in \mathbf{R}^n, \ \ A(\alpha) = A_0 + \sum_{i=1}^{N} \alpha_i A_i, \ \ A_0 \in \mathbf{H}_n, \ \alpha \in \mathbf{\Omega} \tag{9}$$

$$\begin{gathered} \dot{\mu} = \tilde{A}(\alpha)\mu, \quad \mu = B(\alpha)x \in \mathbf{R}^p, \\ \tilde{A}(\alpha) = \begin{bmatrix} A(\alpha) - RC(\alpha) & R \\ C(\alpha)A(\alpha) + G(\alpha)C(\alpha) & -G(\alpha) \end{bmatrix}, \quad B(\alpha) = \begin{bmatrix} I_n \\ C(\alpha) \end{bmatrix}, \end{gathered} \tag{10}$$

where A_i, $i = 0,1,\dots,N$, are given constant matrices, $R \in \mathbf{R}_{n,p-n}$, $C(\alpha) = C_0 + \sum_{i=1}^{N} \alpha_i C_i \in \mathbf{R}_{p-n,n}$ $G(\alpha) = G_0 + \sum_{i=1}^{N} \alpha_i G_i \in \mathbf{R}_{p-n}$ and R, C_i, G_i, $i = 0,1,\dots,N$, are arbitrary matrices. Matrices $\tilde{A}(\alpha)$ and $B(\alpha)$ in (10) satisfy the equality $\tilde{A}(\alpha)B(\alpha) = B(\alpha)A(\alpha)$, $B(\alpha) \in \mathbf{Z}(p,n)$.

Lemma 3.1. Denote $X_i = C_iA_i^+G_iC_i$. For any matrix A_i there exist

(a) integer $p > n$ and real matrices C_i and G_i, such that $X_i = 0_{p-n,n}$;

(b) matrices C_i and G_i (possibly complex), such that $X_i = 0_{p-n,n}$, for arbitrary $p > n$.

Proof. (a) Let $\operatorname{rank}(A_i) = r_i \leq n$, $r = \max\limits_i r_i$, $i = 1,2,\dots,N$, and $p - n \geq r$. Consider the singular value decomposition of matrix $A_i = U_i\Sigma_iV_i^T$, $U_i \in \mathbf{R}_{n,r_i}$, $V_i^T \in \mathbf{R}_{r_i,n}$ and $\Sigma_i \in \mathbf{R}_{r_i}$ is a positive diagonal matrix. When A_i is nonsingular, then U_i and V_i^T are $n \times n$ orthogonal matrices. Let

$$G_i = -C_iY_i, \quad Y_i = [U_i \quad 0_{n,p-n-r_i}] \in \mathbf{R}_{n,p-n}, \quad C_i = \begin{bmatrix} \Sigma_iV_i^T \\ \tilde{C}_i \end{bmatrix}, \quad \tilde{C}_i \in \mathbf{R}_{p-n-r_i,n},$$

where $\tilde{C}_i$ is an arbitrary matrix. Then, $X_i = C_i(A_i^-Y_iC_i) = 0_{p-n,n}$. Since A_i is a real matrix, U_i and V_i^T are also real which proves assertion (a) for p, $2n \geq p > n$. If $p \geq n$ for any C_i of rank n and $G_i = -C_iA_i(C_i^TC_i)^{-1}C_i^T$, one has also $X_i = 0_{p-n,n}$.

(b) Let $G_i = g_i\tilde{G}_i$, where $\tilde{G}_i$ is some nonsingular matrix. Then, $X_i = 0_{p-n,n}$ if and only if $\tilde{G}_i^{-1}C_iA_i^+g_iC_i = 0_{p-n,n}$ which represents a linear, with respect to the unknown matrix C_i, equation and can be put in a compact vector form as

$$K_i\operatorname{vec}(C_i) = 0 \in \mathbf{R}^{n(p-n)}, \qquad \operatorname{vec}(C_i) \in \mathbf{R}^{n(p-n)},$$

$$K_i = A_i^T \otimes \tilde{G}_i^{-1} + g_i I_{n(p-n)}, \qquad K_i \in \mathbf{R}_{n(p-n)},$$

where $X \otimes Y$ and vec($\bullet$) denote the Kronecker product of matrices X, Y and the usual operation that stacks the columns of the matrix argument ($\bullet$) on top of each other, respectively. Denote $K_{i1} = A_i^T \otimes \tilde{G}_i^{-1}$. It is well known that the spectrum of K_{i1} is defined as

$$\sigma(K_{i1}) \equiv \{\lambda_s(A_i)\lambda_k(\tilde{G}_i^{-1}), \quad s = 1, 2, \dots, n, \quad k = 1, 2, \dots, p-n\}.$$

For $g_i = -\lambda_i \in \sigma(K_{i1})$, matrix K_i is singular and $\mathrm{vec}(C_i) \neq 0$ is obtained as an eigenvector for this matrix corresponding to its zero eigenvalue. Depending on the spectrum of A_i, matrices C_i and G_i may be real, or complex. Since p is arbitrary, assertion (b) is proved.

From Lemma 3.1 it follows that matrix $\tilde{A}(\alpha)$ can always be represented as

$$\text{(11.1)} \quad \tilde{A}(\alpha) = \tilde{A}_0 + \sum_{i=1}^{N} \alpha_i \tilde{A}_i + \sum_{i,j=1,\ i<j}^{N} \alpha_i \alpha_j \tilde{A}_{ij}, \qquad \tilde{A}_0 = \begin{bmatrix} A_0 - RC_0 & R \\ C_0 A_0 + G_0 C_0 & -G_0 \end{bmatrix},$$

$$\text{(11.2)} \quad \begin{aligned} \tilde{A}_i &= \begin{bmatrix} A_i - RC_i & 0_{n,p-n} \\ C_0 A_i + C_i A_0 + G_0 C_i + G_i C_0 & -G_i \end{bmatrix}, \\ \tilde{A}_{ij} &= \begin{bmatrix} 0_n & 0_{n,p-n} \\ C_i A_j + C_j A_i + G_i C_j^+ G_j C_i & 0_{p-n} \end{bmatrix}. \end{aligned}$$

Remark 1. As far as asymptotical stability of system (10) is concerned, matrix $\tilde{A}(\alpha)$ is Hurwitz only if $\tilde{A}(0) = \tilde{A}_0$ is Hurwitz, since $\alpha = 0$ is admissible. Having in mind matrix $\tilde{A}$ in (1), it follows that $\sigma(\tilde{A}_0) \equiv \sigma(A_0) \cup \sigma(-C_0 R - G_0)$. By no doubt, there always exist matrices C_0, R and G_0, such that $\tilde{A}_0$ is Hurwitz for any $p > n$.

4. Robust stability. Consider the matrices in (4), where $\tilde{A} = \tilde{A}(\alpha)$, $F = F(\alpha)$, $\bar{\Pi} = \bar{\Pi}(\alpha)$, $\Pi = \Pi(\alpha)$ and matrix (6) with $\bar{L} = \bar{L}(\alpha)$, $\bar{A} = \bar{A}(\alpha)$. Let $\mathbf{\Gamma}$ denotes the set of 2^N vertices of the set $\mathbf{\Omega}$. The next theorem provides a sufficient condition for robust stability of system (9).

Theorem 4.1. Consider (10). Let there exist integers $p > n$, $f \geq 1$, constant matrices R, K, affine parameter-dependent matrices $C(\alpha)$, $G(\alpha)$ satisfying $C_i A_i + G_i C_i = 0$, $i = 1, 2, \dots, N$, $\tilde{A}(0) = \tilde{A}_0 \in \mathbf{H}_p$ and a symmetric matrix

$$\text{(12)} \qquad \Pi(\alpha) = \Pi_0 + \sum_{i=1}^{N} \alpha_i \Pi_i(\alpha) + \sum_{i,j=1,\ i<j}^{N} \alpha_i \alpha_j \Pi_{ij}(\alpha) \in \mathbf{R}_m, \quad m = pf$$

such that $\bar{L}(\alpha)$ in (6) is a negative definite matrix for all $\alpha \in \mathbf{\Gamma}$. Then, the uncertain system (9) is asymptotically stable and the polynomial in α VLF of degree $d = 2(f+1)$, given by

$$\text{(13)} \quad \begin{aligned} v(x, \alpha) &= x^T B^T(\alpha) F^T(\alpha) \Pi(\alpha) F(\alpha) B(\alpha) x \\ &= \sum_{d_1 + \dots + d_N = 0}^{d} \alpha_1^{d_1} \alpha_2^{d_2} \dots \alpha_N^{d_N} v_{d_1 d_2 \dots d_N}^{(} x) = x^T P(\alpha) x \end{aligned}$$

guarantees its robust stability.

Proof. Let the above assumption hold. Having in mind (6), (11) and (12), one has

$$0 > f(y,\alpha) = y^T\bar{L}(\alpha)y = l_{00}(y) + \sum_{i=1}^{N}\alpha_i l_{ii}(y) + \sum_{i,j=1,\, i<j}^{N}\alpha_i\alpha_j l_{ij}(y),$$

$$\forall y \in \mathbf{R}^{m+p}, y \neq 0,\ \forall\alpha \in \mathbf{\Gamma}.$$

Function $f(y,\alpha)$ is multiconvex in α and it is negative on $\mathbf{\Omega}$ if and only if it takes negative values for all $\alpha \in \mathbf{\Gamma}$. Therefore, $\bar{L}(\alpha) < 0$, $\forall\alpha \in \mathbf{\Omega}$. Recall vector $\tilde{\mu}$ in (4). Since $0 > \tilde{\mu}^T\bar{L}(\alpha)\mu = \dot{v}(\mu) = \mu^T[\tilde{A}^T(\alpha)\tilde{P}(\alpha) + \tilde{P}(\alpha)\tilde{A}(\alpha)]\mu$, $\forall\mu \neq 0$, $\forall\alpha \in \mathbf{\Omega}$, $\tilde{P}(\alpha) = F^T(\alpha)\Pi(\alpha)F(\alpha)$ it follows, that $\tilde{P}(\alpha)$ is a nonsingular matrix for all $\alpha \in \mathbf{\Omega}$. For $\alpha = 0 \in \mathbf{\Omega}$, $\tilde{A}(0) = \tilde{A}_0 \in \mathbf{H}_p$ by assumption, and obviously, $\tilde{P}(0) > 0$. By continuity of the eigenvalues of $\tilde{P}(\alpha)$ on $\mathbf{\Omega}$, one has $\tilde{P}(\alpha) > 0$, $\forall\alpha \in \mathbf{\Omega}$. Having in mind that $\mu = B(\alpha)x$, the above scalar inequality can be rewritten as follows:

$$0 > x^T[A^T(\alpha)P(\alpha) + P(\alpha)A(\alpha)]x = \dot{v}(x,\alpha),$$

$$v(x,\alpha) = x^TB^T(\alpha)F^T(\alpha)\Pi F(\alpha)B(\alpha)x > 0$$

for all nonzero vectors x and uncertain vectors $\alpha \in \mathbf{\Omega}$, i.e., system (9) is asymptotically stable and $v(x,\alpha)$ in (13) is a polynomial in α VLF of degree $2(f+1)$, guaranteeing its robust stability.

Conclusions. A new sufficient robust stability condition for a class of uncertain systems has been derived. The proposed approach generalizes some known results considering the same problem and is proved to be less conservative than them.

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